



## TWO FACTOR SHORT RATE GENERIC FIXED INCOME PRICER

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### The theory behind the pricer

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With the development of the Fixed Income markets, the need for multi-factor models describing the short rate dynamics has increased significantly. It is important for their successful application that on one hand closed-form solutions for Caps/Floors and/or Swaptions exist (for calibration purposes) and that on the other hand they are numerically tractable for the pricing of exotic products with American features, path-dependence and coupons depending on several Libor or CMS rates.

We address these problems by offering a two-factor Hull-White type model derived from the HJM framework. This model has two major advantages. It has a very intuitive form and the initial yield curve as an input (by construction). Furthermore it allows for the desired closed-form solutions of Cap/Floor prices as well as a semi-closed form solution of Swaption prices. Based on

this model, we have developed a Generic Fixed Income Pricing Tool (GPT) which allows calibration of the model parameters to ATM Cap volatilities or prices, a Cap volatility/price surface or an ATM Swaption volatility/price surface. Moreover, to price complex claims, we have implemented two reasonable approaches. On the one hand a quadrinomial lattice which deals with the mean reversion – a parameter of the stochastic factors – in a flexible way following the idea of Li, Ritchken and Sankarasubramanian (1995) in the Cheyette tree construction. On the other hand an orthogonal tree construction which overcomes the drawback of the (possibly) negative risk-neutral tree probabilities in the lattice. Additionally, a Monte Carlo Pricer is implemented for all suitable cases. The GPT uses Excel as GUI. The numerical procedures are programmed in Visual.Net (C#) and used as GUI independent libraries.

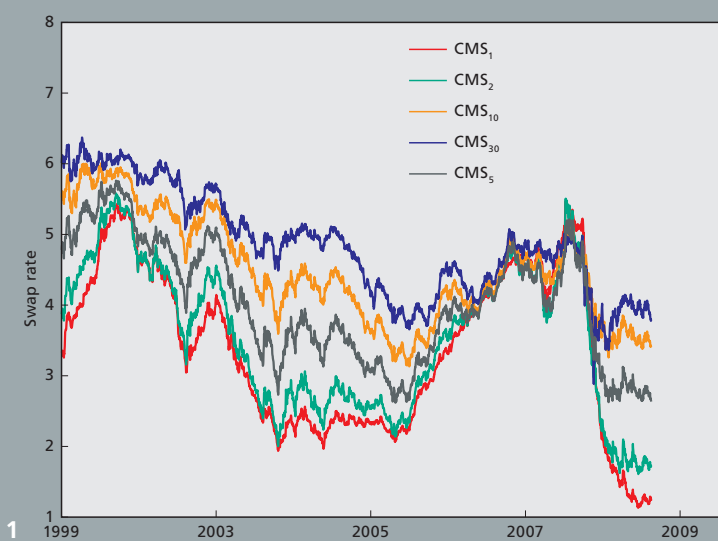
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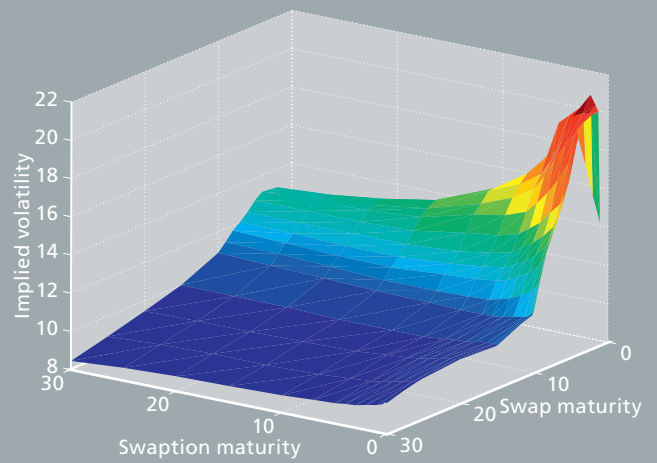


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1 *By the tool generated CMS-rates: The simulations account to the market crisis in 2007*

2 *ATM Swaption volatility surface*

2



### The calibration sheet

For the calibration the user has to supply the relevant market prices and a corresponding yield curve. Afterwards, the user can choose whether to use a local (simplex) solver or a global solver (hybrid adapted simulated annealing). Additionally, since the two factor model can be degenerated to a one

factor Hull-White type Vasicek model, the user can also choose whether to calibrate the one or the two factor model. The ability of the model to degenerate to a well-known one factor model has proven to be very useful for comparison purposes and cross checks.

### Generic description of the coupons and the call/put features

The GPT can be used for pricing products which have only fixed coupons, only floating coupons or both fixed and floating coupons. The floating coupons of the product can depend on one/two Libor rates or one/two CMS rates. Combinations between one Libor and one CMS rate are also possible. Additionally, the GPT allows for a very general form of the floating coupons including e.g. a cap, a floor, multipliers, barriers, indicator functions, etc. The dates of the coupons frequencies can be chosen to be quarterly, semi-annually or annually. In case of non-standard frequencies the user can

input the exact dates by hand. The user can choose between "x business days in advance" or "x business days in arrear" for the fixings of the underlying rates. In addition to the generic structure of the coupons the pricing tool can also price products with both Call and Put features. The call/put dates can be flexibly chosen to be daily (as approximation to American style of options), monthly, quarterly, semi-annually or annually. These dates are calculated using VBA after considering the specified protection period. For non-standard call/put dates the user can input the exact dates manually.

Example list of products that can be priced with the Generic Pricing Tool:

- CMS/Libor Note
- CMS/Libor Floater, Surfer
- CMS/Libor Floating Note
- CMS/Libor (Floored) Participation Note
- CMS/Libor (Dynamic) Spread Note (with a Multiple)
- CMS Spread Cap/Floor
- CMS/Libor (leveraged) Inverse Floater
- CMS Digital Spread Note
- CMS Digital Cap
- Index Plus, Basis Plus

### Path-dependent options

To account for (strong) path-dependence the addition of (at least one) path-dependent variable is required in tree constructions. However, the rule according to which this path-dependent variable is calculated is more "state-of-the-art" than a general rule. Due to this difficulty each product with the described path-dependence has to be considered separately. For that reason we have not added a general pricing routi-

ne for path-dependent products to the GPT. However, with the help of generic programming techniques the pricing of strongly path-dependent products does not pose any additional difficulty to the already developed numerical routines. Examples of such products we have separately priced include a Floatingpfandbrief and a Volatility-linked secured note.