How to solve difficult cutting and packing problems? By semi-infinite optimization!

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Tasks in cutting and packing optimization

The tasks in cutting and packing (C&P) optimization are the following:
- Arrange as many items in a container as possible.
- Arrange all items in a container of minimal size or in as few containers as possible.
- Cut items from one or more containers while minimizing waste.

For specific item and container shapes, there is a huge variety of models and solution methods. But the number of algorithms rapidly decreases as the shape of the items and/or the container becomes more complicated (see Fig. 1 & 2).

From set-theoretic conditions to semi-infinite constraints

There are two types of conditions in any C&P problem:
1) each item \( B_k(x) \) must entirely lie in the container \( C(x) \) (containment conditions):
   \[ B_k(x) \subseteq C(x) \forall k \]
2) the items do not overlap (non-overlapping conditions):
   \[ B_k(x) \cap \text{int}(B_l(x)) = \emptyset \forall k \neq l \]

where some or all objects depend on the decision variables \( x \).

These intractable set-theoretic arrangement conditions can be transferred into semi-infinite constraints in the following way (see [KMS15] and the references therein for details and further reformulations):

Reformulation of 1):
\[
\begin{align*}
\text{If } C(x) = \{ y \in \mathbb{R}^n \mid c_i(x, y) \leq 0, i \in I \}, \text{ then } B_k(x) \subseteq C(x) \\
\Rightarrow c_i(x, y) \leq 0 \forall y \in B_k(x), i \in I.
\end{align*}
\]

Reformulation of 2):
\[
\begin{align*}
B_k(x) \cap \text{int}(B_l(x)) = \emptyset \\
\Rightarrow \exists \eta \neq 0 \text{ and } \beta : \\
\eta^T y - \beta \leq 0 \forall y \in B_k(x) \\
\eta^T x - \beta \geq 0 \forall x \in B_l(x).
\end{align*}
\]

Applied to gemstone cutting

In gemstone cutting an irregularly shaped raw gem having surface flaws and being interspersed with inclusions must be cut into blanks such that the total value of the manufacturable precious gems is maximized (see Fig. 8). We considered three of the four price-setting criteria, the so-called the 4 Cs: carat/volume, clarity, cut, and color (MM12).

The main challenges are:
- irregular raw gem and defects,
- wide range of different shapes and cuts,
- size-dependent faceting, and
- taking aesthetic sensibility into account.

By improving as well as developing new semi-infinite optimization techniques we demonstrated, that realistic problem instances can be solved on a standard PC in reasonable time (see Fig. 9 & KMS15) for details).

The work resulted not only in many publications, but also in a software product (see Fig. 10) and in the first fully automated gemstone production process (see Fig. 11) [ITWM, FP09].

Summary and outlook

On the one hand, semi-infinite optimization allows modelling and solving of difficult C&P problems in the first place. And on the other hand it provides a unified approach to solve C&P problems in general. Furthermore, by this way one can also solve other geometry optimization problems like layout or positioning problems.

Future topics of research are:
- freeform (water/laserjet) cutting (see Fig. 12),
- computation of globally optimal arrangements, and
- automation of raw gem sectioning.

References (selected)


